Coimisiún na Scrúduithe Stáit
State Examinations Commission

LEAVING CERTIFICATE EXAMINATION

APPLIED MATHEMATICS – HIGHER LEVEL

DURATION – 2 HOURS AND 30 MINUTES

Five questions to be answered. All questions carry equal marks.

A *Formulae and Tables* booklet may be obtained from the Superintendent.

Take the value of g to be 9.8 m s^{-2}.

Marks may be lost if necessary work is not clearly shown.

Marks may be lost for omission of correct units with numerical answers.

Diagrams are generally not drawn to scale.

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1. **(a)** Two cars, A and B, travel along a straight level road in opposite directions. A passes point *P* with speed 4 m s^{-1} and uniform acceleration 2 m s^{-2}. Three seconds later B passes point *Q* with speed 5 m s^{-1} and uniform acceleration 4 m s^{-2}.

The distance from *P* to *Q* is 1143 m.

The cars meet *t* seconds after A passes *P*.

(i) Find the value of *t*.

(ii) Find the distance from *P* to the meeting point.

(iii) Find the distance between the cars when A is 160 m from the meeting point, before the cars meet.

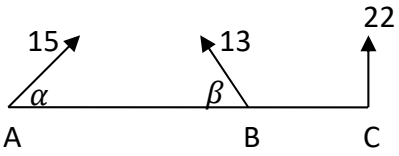
(b) An object falls vertically, from rest, from a height *h* metres. It travels $\frac{15}{64}h$ metres during its final second of motion before hitting the ground.

(i) Find the time it takes to fall to the ground.

(ii) Find the value of *h*.

2. **(a)** Ship A is 42 km due west of ship B.

Ship B is 25 km due west of ship C.



A is moving east α° north at a constant speed of 15 km per hour. B is moving west β° north at a constant speed of 13 km per hour. C is moving due north at a constant speed of 22 km per hour.

$\text{Tan}\,\alpha = \frac{4}{3}$ and $\tan\beta = \frac{12}{5}$.

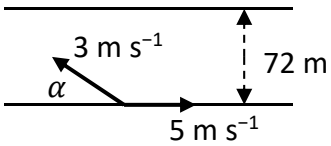
A and B meet after *t* hours.

(i) Find the value of *t*.

(ii) At the instant A and B meet, how far is C from the meeting point?

(b) A boat travels at 3 m s^{-1} in still water.

The boat is aimed at an angle α to the upstream direction of a river which is 72 m wide and flows at 5 m s^{-1} parallel to the straight banks.



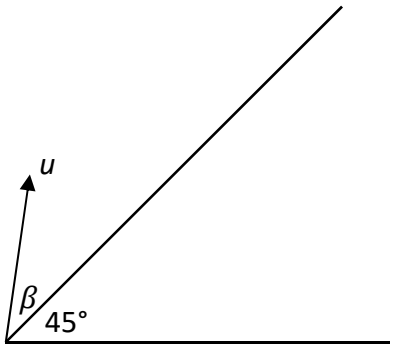
(i) Find the value of α if the boat is to land on the opposite bank as little downstream as possible.

(ii) Find the time taken to cross the river under these conditions.

3. **(a)** A particle is projected from a point on horizontal ground. The speed of projection is 14 m s^{-1} at an angle α to the horizontal.

Find the two values of α that will give a range of 10 m.

(b) A plane is inclined at an angle 45° to the horizontal. A particle is projected up the plane with speed *u* at an angle β to the plane.



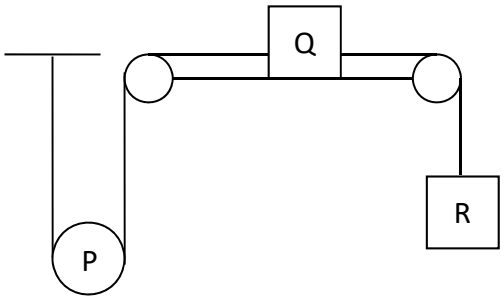
The plane of projection is vertical and contains the line of greatest slope.

(i) Find the time of flight of the particle.

(ii) Find the range of the particle on the inclined plane.

(iii) If the particle strikes the plane horizontally, find β.

4. **(a)** A taut light inelastic string is fixed at one end and passes under a moveable pulley, P, of mass 4 kg which hangs vertically. The other end of the string is attached to Q, a mass of 4 kg which lies on a rough horizontal surface.



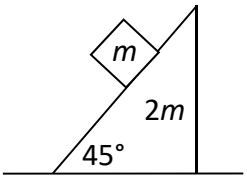
A second inelastic string connects Q to R, a mass of 10 kg which hangs vertically.

The fixed pulleys are smooth and light and the coefficient of friction between Q and the surface is $\frac{1}{2}$.

The system is released from rest.

Find the accelerations of P, Q and R in terms of *g*.

(b) A smooth wedge of mass 2*m* and slope 45° rests on a smooth horizontal surface. A particle of mass *m* is placed on the smooth inclined face of the wedge. The system is released from rest.



(i) Show, on separate diagrams, the forces acting on the particle and on the wedge.

(ii) Find the acceleration of the wedge.

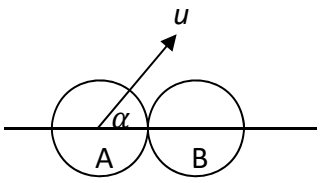
(iii) Find the acceleration of the particle relative to the wedge.

5. **(a)** A smooth sphere, P, of mass 3*m* collides directly with another smooth sphere, Q, of mass 5*m*. P and Q are moving in opposite directions before impact with speeds 4*u* and 2*u* respectively. The coefficient of restitution for the collision is *e*.

(i) Find the speed of P and the speed of Q after impact in terms of *u* and *e*.

(ii) If P and Q are moving in the same direction after impact, show that $0 \leq e < \frac{1}{15}$.

(b) A smooth sphere, A, of mass *m* collides obliquely with another smooth sphere, B, of mass *m*.



Before impact, A is moving with speed *u* at an angle α to the line of centres of the spheres, where $0° < \alpha < 45°$.

B is at rest before the impact.

The coefficient of restitution for the collision is *e*.

(i) Find the speed of A and the speed of B after impact in terms of *u*, *e* and α.

(ii) Given that A is deflected through angle α because of the collision, show that $\tan^2 \alpha = e$.

6. **(a)** A particle moves with simple harmonic motion with centre *P*. When its displacement from *P* is 3 m the particle has speed 8 m s^{-1} and when its displacement from *P* is 4 m the particle has speed 6 m s^{-1}.

(i) Find the amplitude of the motion.

(ii) Find the period.

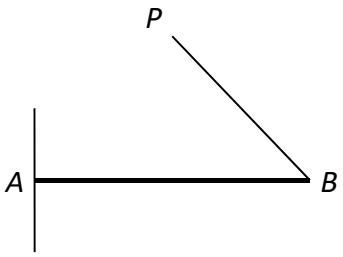
(iii) Find the time taken to travel from an extreme point to a point mid-way between the extreme point and *P*.

(b) A particle of mass *m* is suspended vertically from a fixed point *O* by a light inelastic string of length *d* metres.

The particle is projected horizontally with speed *u*, where $u^2 = 4gd$.

Show the string goes slack when it makes an angle $\cos^{-1}\frac{2}{3}$ with the upward vertical through *O*.

7. **(a)** A uniform rod, *AB*, of weight 90 N and length 6 m is smoothly hinged at *A* to a vertical wall.



A light inelastic string has one end attached to *B* and the other end is tied to a fixed point *P*.

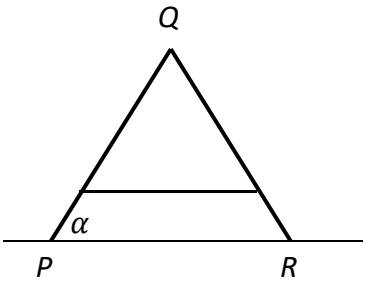
P is 4 m vertically above the mid-point of *AB*.

The rod lies horizontally and in equilibrium.

(i) Find the tension in the string.

(ii) Find the magnitude of the reaction at the hinge.

(b) Two uniform rods, *PQ* and *QR*, are smoothly hinged at *Q* and rest in a vertical plane with *P* and *R* on a smooth horizontal surface.



The rods are equal in length and each has weight *W*.

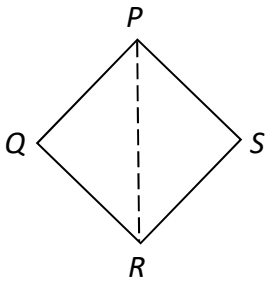
A horizontal string has its ends attached to points a quarter way up each rod.

$|\angle QPR| = \alpha$

Show that the tension in the string is $\frac{2W}{3\tan\alpha}$.

8. **(a)** Prove that the moment of inertia of a uniform square lamina, of mass *m* and side 2*a*, about an axis through its centre parallel to one of its sides is $\frac{1}{3}ma^2$.

(b) A uniform square lamina, of mass *m* and side 2*a*, has vertices *P*, *Q*, *R* and *S* and *PR* is a diagonal.



A particle of mass *m* is attached at *R*.

The compound body is freely pivoted at *P* and performs small oscillations in a vertical plane.

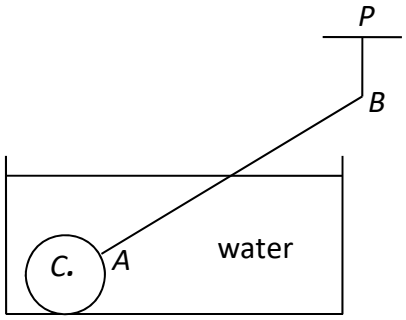
(i) Show that the periodic time is $\frac{8\pi}{3}\sqrt{\frac{\sqrt{2}a}{g}}$.

(ii) If the length of the equivalent simple pendulum is $\frac{8\sqrt{2}}{9}$, find the value of *a*.

9. **(a)** A buoy in the shape of a hollow spherical shell of external radius 1 m and internal radius *r* floats in water with 74% of its volume immersed. The density of the material of the shell is 1280 kg m^{-3} and the density of water is 1000 kg m^{-3}.

Find the value of *r*.

(b) A thin uniform rod *AB* of weight 2*W* and length 6*r* is attached to a solid sphere of weight *W* and centre *C*. The radius of the sphere is *r*.



The points *C*, *A* and *B* are collinear. The relative density of both the rod and the sphere is 10.

The sphere is completely immersed in water and rests on the bottom of the container. Half the rod is immersed in the water.

The rod lies inclined to the vertical and a light inelastic vertical string is attached at *B* and to a fixed point *P*.

Find, in terms of *W*,

(i) the tension in the string

(ii) the reaction between the sphere and the bottom of the container.

[Density of water = 1000 kg m^{-3}]

10. **(a)** **(i)** Solve the differential equation

$$(1 + t^2)\frac{dr}{dt} = 1$$

given that $r = 0$ when $t = \frac{\pi}{4}$.

(ii) If

$$\frac{dy}{dx} = (y + 4)\cos^2 3x$$

and $y = -3$ when $x = 0$, find the value of y when $x = \frac{\pi}{6}$.

(b) A particle is projected horizontally along a smooth horizontal surface with initial speed 80 m s^{-1}. The particle has a retardation of $\frac{v}{100}$ m s^{-2}, where v is the speed.

Find

(i) the speed of the particle after *t* seconds

(ii) the distance travelled in *t* seconds

(iii) the speed v in terms of the distance travelled, *s*.

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Leaving Certificate Examination – Higher Level

Applied Mathematics

Duration: 2 hours and 30 minutes